

Theory overview on amplitude analyses with charm decays

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This contribution about amplitude analyses in multibody hadronic charm decays deals with some attempts to introduce theoretical constraints. Different effective hadronic formalism approaches are mentioned. A recent work, based on a basic weak interaction process and a Chiral unitary model to account for the final state interaction, is described in details for the $f_0(980)$ production in $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$ and $D_s^+ \rightarrow \pi^+ K^+ K^-$ decays. Within the framework of the diagrammatic approach and flavor symmetry, a global analysis of two-body D decays into a vector meson and a pseudoscalar meson is presented. A quasi-two-body QCD factorization model for D decays into three mesons and its recent application to $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ is outlined. For processes with final-state pions and kaons and as an alternative to the sum of Breit-Wigner amplitudes, often used in experimental Dalitz-plot analyses, amplitude parametrizations, in term of unitary $\pi\pi$, πK and $K\bar{K}$ form factors, are proposed. These parametrizations are derived from quasi-two-body factorization models.

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1. Introduction

There is an impressive set of hadronic multibody decay data for D^0 , D^+ and D_s^+ decays [1, 2, 3]. The Dalitz plots are characterized by an accumulation of events displaying the presence of meson resonances and their interferences [4, 5, 6, 7, 8]. The Standard model (SM) predicts null CP asymmetries and some deviation could be a signal of physics beyond SM [9, 10]. Furthermore the study of D^0 - \bar{D}^0 mixing might indicate the presence of new physics contributions [11, 12, 13, 14]. Multibody hadronic decays of $D_{(s)}$ mesons consist of a weak process, microscopic quark flavor changing process like $c \rightarrow d$ or $c \rightarrow s$ via the W meson interaction, followed by hadronization and final state meson-meson strong interaction processes. Basic amplitude analyses are usually performed via the isobar model or sum of relativistic Breit-Wigner terms representing the different possible implied resonances plus a non-resonant background: can one go beyond?

Section 2 is devoted to some final state interaction (FSI) studies, section 3 to a diagrammatic approach and flavor symmetry for $D \rightarrow VP$ decays ($V, P \equiv$ vector, pseudoscalar mesons), section 4 to a quasi-two-body QCD factorization model for D decays into three mesons and section 5 to amplitude parametrizations based on quasi-two-body factorization and to some conclusions.

2. Final-state interaction constraints

2.1 Different effective hadronic formalism approaches

The $K^- \pi^+$ FSI in the $D^+ \rightarrow K^- \pi^+ \pi^+$ data of the E791 Collaboration has been the subject of many studies using different effective hadronic formalism approaches [15, 16, 17, 18]. In Ref. [15] it is shown that final state interactions are important in shaping the Dalitz plot and that several weak and hadronic processes are required. In Ref. [16] the theoretical treatment of this decay includes a rich dynamic behavior that mix weak and strong interactions in a non trivial way. The authors of Ref. [17] assume the dominance of the weak vector current together with a Chiral effective Lagrangian and phenomenological form factors. S. X. Nakamura [18] has performed a coupled-channel analysis of pseudo-data generated from the isobar model of the E791 Collaboration. The authors of Ref. [19] have studied the $D^+ \rightarrow K^+ K^- K^+$ process with a multi-meson model as an alternative to isobar model, with free parameters predicted by the theory to be fine-tuned by a fit to data.

2.2 Basic weak interaction plus Chiral unitary approach

The study of $f_0(980)$ production in $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$ and $D_s^+ \rightarrow \pi^+ K^+ K^-$ decays performed by the authors of Ref. [20] is described below. For the sake of completeness the main steps of their derivation is reproduced here. They start from the Cabibbo favored $c \rightarrow s \bar{d} u$ flavor changing process. Then the $c \bar{s}$ pair of the D_s^+ decays into a $\bar{d} u$ (which hadronizes into a π^+) and a $s \bar{s}$ pair. Insertion, in the $s \bar{s}$ pair, of a $q \bar{q}$ with the quantum numbers of the vacuum, $\bar{u} u + \bar{d} d + \bar{s} s$, leads then, via hadronization, to the production of two pseudoscalar mesons. This dominant process is depicted in Fig. 1 of Ref. [20]. To find out the meson-meson components in the $s \bar{s}$ pair one can define the following $q \bar{q}$ M matrix:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}, \quad (2.1)$$

which satisfies

$$M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s) \quad (2.2)$$

In the standard $\eta - \eta'$ mixing [21] the matrix M is related to [22]

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \quad (2.3)$$

Neglecting η' contribution, its mass being too large, one has:

$$s\bar{s}(\bar{u}u + \bar{d}d + \bar{s}s) \equiv (\Phi \cdot \Phi)_{33} = K^- K^+ + \bar{K}^0 K^0 + \frac{1}{3}\eta\eta, \quad (2.4)$$

which are the produced states before FSI. After rescattering the $K^+ K^-$ pair can produce $\pi^+ \pi^-$ and/or $K^+ K^-$ pairs. The D_s^+ decay width into a π^+ and two mesons, labelled, $\Gamma_{P^+ P^-}$, where $P^+ P^- \equiv K^+ K^-$ or $\pi^+ \pi^-$, satisfies

$$\frac{d\Gamma_{P^+ P^-}}{dM_{inv}} = \frac{1}{(2\pi)^3} \frac{p\pi \tilde{p}_P}{4M_{D_s}^2} |T_{P^+ P^-}|^2, \quad (2.5)$$

with,

$$T_{K^+ K^-} = V_0 \left(1 + G_{K^+ K^-} t_{K^+ K^- \rightarrow K^+ K^-} + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0 \rightarrow K^+ K^-} + \frac{2}{3} \frac{1}{\sqrt{2}} G_{\eta\eta} t_{\eta\eta \rightarrow K^+ K^-} \right), \quad (2.6)$$

$$T_{\pi^+ \pi^-} = V_0 \left(G_{K^+ K^-} t_{K^+ K^- \rightarrow \pi^+ \pi^-} + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0 \rightarrow \pi^+ \pi^-} + \frac{2}{3} \frac{1}{\sqrt{2}} G_{\eta\eta} t_{\eta\eta \rightarrow \pi^+ \pi^-} \right). \quad (2.7)$$

The function G_l is the loop function,

$$G_l(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p-q)^2 - m_1^2 + i\epsilon} \frac{1}{q^2 - m_2^2 + i\epsilon}, \quad (2.8)$$

m_1, m_2 being the meson masses in loop l . The integral on q^0 is analytical and a cut-off, $|\mathbf{q}_{max}| = 600 \text{ MeV}/c$ is needed in the integral on \mathbf{q} in order to reproduce the experimental amplitudes. The $t_{i \rightarrow j}$ matrices are obtained by solving the coupled-channel Bethe-Salpeter equation

$$t_{i \rightarrow j}(s) = V_{ij}(s) + \sum_{l=1}^5 V_{il}(s) G_l(s) t_{l \rightarrow j}(s), \quad (2.9)$$

where the indices i, j, l running from 1 to 5 denote the different channels: 1 for $\pi^+\pi^-$, 2 for $\pi^0\pi^0$, 3 for K^+K^- , 4 for $K^0\bar{K}^0$, and 5 for $\eta\eta$. The kernel V_{ij} are the tree-level transition amplitudes built from phenomenological Lagrangians in Ref. [23].

Adjusting V_0 and comparing theoretical amplitudes $T_{K^+K^-}$ and $T_{\pi^+\pi^-}$ with those available from the experimental data [24, 25] leads to a fair agreement as can be seen in Figs. 2, 3 (and 4) of Ref. [23]. Invariant mass distributions for $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ and $D_s^+ \rightarrow \pi^+K^-K^+$ are shown in Fig. 5 of Ref. [23]. The $f_0(980)$ signals in the spectra are (up to a global common normalization factor) predictions of the Chiral Unitary approach with no free parameters. The mechanism displayed above was used in Ref. [26] to obtain branching ratios for $a_0(980)$ and $f_0(980)$ production in good agreement with experiment. An interesting issue will be the study of the $\pi^+\pi^0\eta$ decay mode which generates the $a_0(980)$ and can lead to informations on possible $f_0(980)$ and $a_0(980)$ mixing.

3. Diagrammatic approach for D decays into a vector and a pseudoscalar meson

3.1 $D \rightarrow VP$ decays within SU(3) flavor symmetry

The c quark mass, m_c , being too high to apply Chiral perturbation theory and too light to use heavy quark expansion approaches, one can use a diagrammatic approach with flavor-flow diagrams classified according to the topologies of weak interactions with all strong interaction effects included. This model-independent analysis, based on flavor SU(3) symmetry, determines topological amplitudes allowing to specify the relative importance of different underlying decay mechanisms. In such an approach, introduced by L. L. Chau [27], the conceivable topologies of weak interactions are shown in Fig. 1 of Ref. [28].

We describe here some results of the recent work of Ref. [29] in which all two-body charmed meson decays $D \rightarrow VP$ are studied in this diagrammatic framework. There, within SU(3) flavor symmetry, only four types of amplitudes exist: color-allowed T , color-suppressed C , W -exchange E , and W -annihilation A . Subscript P or V to each amplitude, *e.g.*, $T_{P(V)}$, denote the amplitude in which the spectator quark goes to the pseudoscalar or vector meson in the final state. These two kinds of amplitudes do not have *a priori* any obvious relationship. All the flavor amplitude magnitudes and their associated strong phases are then extracted through a fit on existing experimental branching fractions.

3.2 Fit on branching fractions of $D \rightarrow VP$ decays

The 8 complex amplitudes, $T_{P(V)}, C_{P(V)}, E_{P(V)}, A_{P(V)}$ (15 real parameters, T_V chosen to be real) are determined by performing a χ^2 fit of 16 experimental branching fractions for Cabibbo-favored D^0 and $D_{(s)}^+$ decays proportional to the Cabibbo-Kobayashi-Maskawa (CKM) factors $V_{cs}^*V_{ud} \sim \mathcal{O}(1)$. All the data are extracted from the Particle Data Group [30] but the branching fraction $\mathcal{B}_{\rho^+\eta'}$ is taken from Ref. [31]. The determination of $D_s^+ \rightarrow \pi^+\rho^0$ using results from Refs. [25, 30] (see Ref. [29]) allows the extraction of the $A_{P(V)}$ amplitudes. Among the several solutions found by the authors there is one favored, named (A1). The results of the fits are shown in the Table II of Ref. [29]. Comparison is made in particular with the pole model of Ref. [32] which is built using generalized factorization and addition of poles in annihilation diagrams.

The Cabibbo-favored amplitudes, resulting from the branching fraction fit results (solution A1) are shown in Table 1.

$ T_P $	δ_{T_P}	$ C_V $	δ_{C_V}	$ C_P $	δ_{C_P}	$ E_V $
$8.46^{+0.22}_{-0.25}$	57^{+35}_{-41}	$4.09^{+0.16}_{-0.25}$	-145^{+29}_{-39}	$4.08^{+0.37}_{-0.36}$	-157 ± 2	$1.19^{+0.64}_{-0.46}$
δ_{E_V}	$ E_P $	δ_{E_P}	$ A_P $	δ_{A_P}	$ A_V $	δ_{A_V}
-85^{+42}_{-39}	3.06 ± 0.09	98 ± 5	$0.64^{+0.14}_{-0.27}$	152^{+48}_{-50}	$0.52^{+0.24}_{-0.19}$	122^{+70}_{-42}

Table 1: Cabibbo-favored amplitudes resulting from the branching fraction fit results [29] (solution A1) for a η - η' mixing angle of 43.5° . Units: 10^{-6} , strong phases in degrees. $|T_V| = 4.21^{+0.18}_{-0.19}$.

The modulus of color-allowed tree T_P amplitude is the largest. The moduli of color-allowed tree T_V , color-suppressed tree $C_{V(P)}$ and W -exchange E_P are of the same magnitude. The moduli of the W -annihilation $A_{P(V)}$ amplitudes are the smallest.

Branching fraction predictions, with no flavor SU(3) breaking, for singly Cabibbo-suppressed decays proportional to $V_{cd}^* V_{ud} \sim \mathcal{O}(\lambda)$ and to $V_{cs}^* V_{us} \sim \mathcal{O}(\lambda)$ with $\lambda = 0.22543$ (CKMfitter, see Ref. [14]) can be seen in Table III of Ref. [29] and predictions for doubly Cabibbo-suppressed decays (no SU(3) breaking) proportional to $V_{cd}^* V_{us} \sim \mathcal{O}(\lambda^2)$ in Table IV. The predictions for the doubly Cabibbo-suppressed channels are in good agreement with data but the singly Cabibbo-suppressed ones have some flavor SU(3) symmetry breaking effects.

3.3 Concluding remarks on this $D \rightarrow VP$ study

Exact flavor SU(3) describes reasonably well the available data. If T and C amplitudes are factorizable, the effective Wilson coefficients $a_{1,2}$, $|a_2/a_1|$ and $\arg(a_2/a_1)$ (see next section) can be extracted from Cabibbo-favored $D^+ \rightarrow \bar{K}^{*0} \pi^+$ and $\bar{K}^0 \rho^+$ [solution (A1)]. The results are shown in Table VII of Ref. [29]. SU(3) symmetry breaking in color-allowed T and color-suppressed C tree amplitudes is needed in general to have a better agreement with experiment. Nevertheless, the exact flavor SU(3)-symmetric approach alone is adequate to provide an overall explanation for the current data. The impact of this symmetry on $D \rightarrow PP$ decays has been presented in this workshop by P. Santorelli [33]. Within the diagrammatic approach one should quote the validity study of flavor SU(3) [34].

4. Factorization approach for hadronic three-body D decays

4.1 Quasi-two-body factorization for $D^0 \rightarrow K_S^0 \pi^+ \pi^-$

QCD factorization beyond naïve factorization, expansion in α_s (strong coupling constant) and $1/m_b$ (m_b b -quark mass), applies with success to charmless nonleptonic two-body B decays [35]. In D decays, $m_c \sim m_b/3$ leads a priori to significant corrections to the factorized results and factorization is more a phenomenological approach, based on the seminal work by Bauer, Stech and Wirbel [36]. It is then applied successfully to D decays, treating Wilson coefficients as phenomenological parameters to account for non-factorizable corrections. A part from a recent attempt to extend the framework of QCD factorization to non-leptonic B decays into three light mesons [37], so far there exists no factorization theorem for three-body decays. However there are important contributions from intermediate resonances as $\rho(770)$, $K^*(892)$ and $\phi(1020)$ and three-body decays may

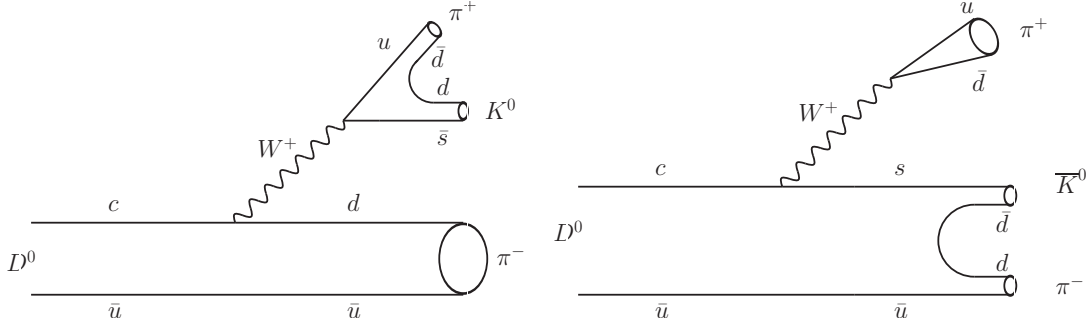


Figure 1: Microscopic quark tree diagrams for doubly Cabibbo-suppressed (left) and Cabibbo-favored (right) amplitudes

be considered as quasi-two-body decays. One makes the hypothesis that two of the three final-state mesons form a single state originating from a quark-antiquark pair. This leads to a quasi-two-body final state to which the factorization procedure is applied.

Within this framework we report on the Dalitz plot studies of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays performed in Ref. [38]. There is no penguin (W -loop diagram) in this decay and the weak effective Hamiltonian reads

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_{i=1,2} C_i(\mu) O_i(\mu) + h.c., \quad (4.1)$$

where V_{CKM} represents the quark mixing couplings, $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ the Fermi coupling, $C_i(\mu)$ the QCD Wilson coefficients arising from W exchange and μ the renormalization scale with $\mu \sim m_c = 1.3 \text{ GeV}$. The left-handed quark current-current operators O_1 reads

$$O_1 = j_1 \otimes j_2, j_1 = \bar{s}_\alpha \gamma^\nu (1 - \gamma^5) c_\alpha \equiv (\bar{s}c)_{V-A}, j_2 = \bar{u}_\beta \gamma^\nu (1 - \gamma^5) d_\beta \equiv (\bar{u}d)_{V-A}, \quad (4.2)$$

α and β being color indices. The operator O_2 has a similar expression.

In the amplitude and at leading order in α_s , the following real effective QCD coefficients $a_1(m_c)$ and $a_2(m_c)$ will appear,

$$a_1(m_c) = C_1(m_c) + \frac{C_2(m_c)}{N_C}, \quad a_2(m_c) = C_2(m_c) + \frac{C_1(m_c)}{N_C}, \quad (4.3)$$

$N_C = 3$ being the number of colors (from now on $a_i(m_c) \equiv a_i, i = 1, 2$). The use of the Operator Product Expansion and of the fact that the W mass is large leads to the two-body factorization approximation,

$$\langle M_1 M_2 | O_i(\mu) | D^0 \rangle = \langle M_1 | j_1 | 0 \rangle \langle M_2 | j_2 | D^0 \rangle + \text{higher order corrections}. \quad (4.4)$$

In the quasi-two-body approximation, the final state $\bar{K}^0 \pi^+ \pi^-$ is assimilated to the two-body states $[\bar{K}^0 \pi^\pm]_L \pi^\mp$ and $\bar{K}^0 [\pi^+ \pi^-]_L$ where the meson-meson pair can be in $L = S, P$ and D states. The mesons $M_{1,2}$ of the factorization equation (4.4) are then $M_1 = [\pi^+ K^0]_L, M_2 = \pi^-$ or $M_1 = \bar{K}^0, M_2 = [\pi^+ \pi^-]_L$. In the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays the possible c quark flavor changing are $c \rightarrow$

$s\bar{u}\bar{d}$ leading to Cabibbo-favored (CF) amplitudes $\propto V_{CKM} = V_{cs}^* V_{ud} \equiv \Lambda_1$ and $c \rightarrow du\bar{s}$ for doubly Cabibbo-suppressed (DCS) amplitudes $\propto V_{CKM} = V_{cd}^* V_{us} \equiv \Lambda_2$. Here, besides tree amplitudes, there are annihilation ones arising from W exchange between the quarks c and \bar{u} of the D^0 . The interested reader will find the detailed expressions of the different amplitudes obtained applying the above quasi-two-body factorization approximation to the $\langle \bar{K}^0 \pi^- \pi^+ | H_{eff} | D^0 \rangle$ matrix element. Let us exemplify some terms entering these amplitudes. In the DCS tree amplitude (see Eq. (10) of Ref. [38]) for $L = S$ one encounters the contribution of a term like (see left diagram of Fig. 1)

$$\frac{G_F}{2} \Lambda_2 a_1 \langle \pi^- | (\bar{d} c)_{V-A} | D^0 \rangle \cdot \langle [K^0 \pi^+]_S | (\bar{u} s)_{V-A} | 0 \rangle, \quad (4.5)$$

where the first matrix element is related to the $D^0 \rightarrow \pi$ transition form factor and the second one to the scalar $K\pi$ form factor. In the CF tree amplitude (see Eq. (6) of Ref. [38]) for $L = S$ one has the contribution of a term as (see right diagram of Fig. 1)

$$\frac{G_F}{2} \Lambda_1 a_1 \langle \pi^+ | (\bar{u} d)_{V-A} | 0 \rangle \cdot \langle [\bar{K}^0(p_0) \pi^-]_S | (\bar{s} c)_{V-A} | D^0(p_{D^0}) \rangle, \quad (4.6)$$

where the first matrix element is related to the π decay constant and the second one to the scalar $D^0 \rightarrow K\pi$ transition form factor. Its evaluation is less straightforward, it could be experimentally evaluated from semi-leptonic processes such as, $D^0 \rightarrow K^- \pi^+ \mu^+ \mu^-$ [39, 40]. Here assuming this transition to proceed through the dominant intermediate resonance $K_0^*(1430)$, it can be written in terms of the $K\pi$ scalar form factor. The corresponding amplitude given by Eq. (13) of Ref. [38] is

$$T_{[\bar{K}^0 \pi^-]_S \pi^+}^{CF}(s_0, s_-, s_+) = -\frac{G_F}{2} a_1 \Lambda_1 \chi_1 (m_{D^0}^2 - s_-) f_\pi F_0^{D^0 K_0^*(1430)^-}(m_\pi^2) F_0^{\bar{K}^0 \pi^-}(s_-), \quad (4.7)$$

where $s_\pm = (p_{\pi^\pm} + p_{K^0})^2 = m_{\pm}^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2 = m_0^2$. The scalar $K\pi$ form factor $F_0^{\bar{K}^0 \pi^-}(s_-)$ includes the contributions of the $K_0^*(800)$ and $K_0^*(1430)$ resonances. The factor χ_1 , related to the strength of this scalar form factor is taken as a complex constant to be fitted. It can be estimated from the $K_0(1430)$ properties [38]. Here f_π is the pion decay constant. Following Ref. [41] the value of 0.48 is used for the D^0 to $K_0^*(1430)$ transition form factor $F_0^{D^0 K_0^*(1430)^-}(m_\pi^2)$.

What we have shown just above for amplitudes with $K\pi$ final state pair in S wave, works also for the P -wave case and for amplitudes with the $\pi^+ \pi^-$ final state pair in S and P wave. So, amplitudes with the $\pi^+ \pi^-$ ($K\pi$) final state pair in S and P wave are described in terms of scalar and vector $\pi\pi$ ($K\pi$) form factors. Form factors with final meson pair in D wave are represented by relativistic Breit-Wigner formulae. The 13 tree-amplitudes and 14 W -exchange ones derived in Ref. [38] can be recast into 10 amplitudes. Summary of these CF and DCS, amplitudes associated to the different quasi two-body channel together with the contributing dominant resonances are listed in Table 1 of Ref. [38].

4.2 Form factors

It can be shown from field theory and using dispersion relations [42] that meson-meson form factors can be calculated exactly, using Muskhelishvili-Omnès equations, if one knows the meson-meson strong interactions at all energies. The details and corresponding references of the unitary scalar and vector $K\pi$ and $\pi\pi$ form factors used in the best fits of the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot

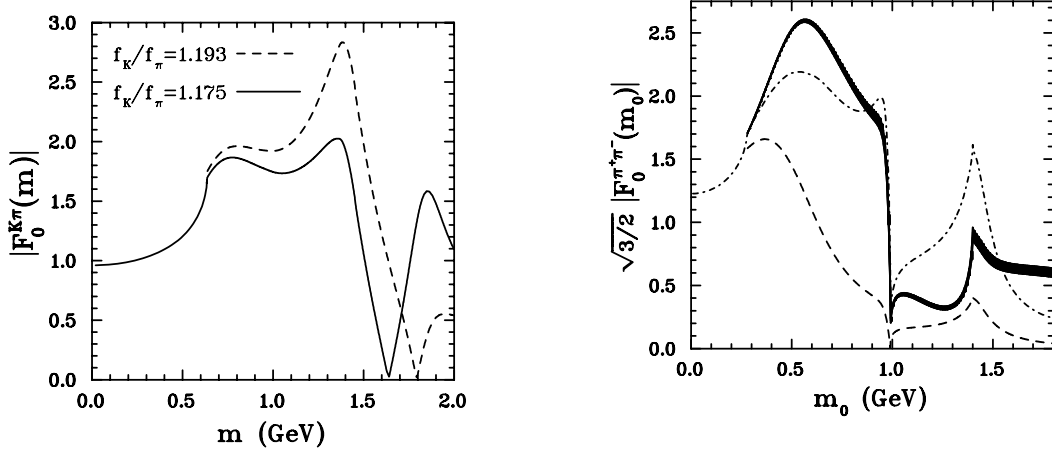


Figure 2: Left panel: modulus of the scalar $K\pi$, $F_0^{K\pi}$, form factor as a function of the effective $K\pi$ mass for two values of the f_K/f_π ratio (f_K being the kaon decay constant). Right panel: modulus of the scalar $\pi\pi$, $F_0^{\pi\pi}$, form factor as a function of the effective $\pi\pi$ mass, dark band variation within the parameter errors of the fit to Belle data [43], dashed line same form factor but different parameters (Ref. [44]), dot-dash line is the result of Ref. [45] using Muskhelishvili-Omnès equations.

performed in Ref. [38] can be found in this reference. We just plot here, in Fig. 2, the modulus of the scalar $K\pi$ ($F_0^{K\pi}$, left panel) and $\pi\pi$ ($F_0^{\pi\pi}$, right panel) form factors as function of the $K\pi$ (m_\pm) and $\pi\pi$ (m_0) effective masses. The scalar $K\pi$ form factor is characterized by two bumps arising from the $K_0^*(800)$ and $K_0^*(1430)$ contributions and the $\pi\pi$ form factor by a dip coming from the $f_0(980)$ and two bumps from the $f_0(500)$ and $f_0(1400)$.

4.3 Dalitz-plot fit

The Dalitz plot distribution of the best fit to the Belle data [43] is shown in Figs. 9 of Ref. [38]. This distribution reproduces well that of Belle. The corresponding input parameters and the obtained 33 free parameters are given in section IV and in Table II of Ref. [38], respectively. The Dalitz plot shows a rich interference pattern with the dominance of the $K^*(892)$.

Branching fraction (Br), listed in Table V of Ref. [38], compare well with those of Belle's analysis. Their sum, equal to 133 %, shows the importance of interferences. The largest Br come from the amplitudes, \mathcal{M}_1 [contributions of $K_0^*(800)^-$, $K_0^*(1430)^-$] with a Br of 25. %, \mathcal{M}_2 [contribution of $f_0(500)$, $f_0(980)$, $f_0(1400)$] with a Br of 16.9 %, \mathcal{M}_3 [contributions of $K^*(892)^-$] with a Br of 62.7 %) and \mathcal{M}_4 [contributions of $\rho(770)^0$] with a Br of 21.9 %. The sum of the Belle Br

with the final $\pi^+\pi^-$ pair state in S wave, equal to 18.6%, is close the Br of \mathcal{M}_2 , equal to 16.9 %. The annihilation (W exchange) contributions can be important.

5. Concluding remarks

5.1 Amplitude parametrizations

As a sound alternative to the simplistic and widely used isobar model, explicit amplitude parametrizations, that can be readily implemented in experimental analysis, are suggested in Refs. [46, 47] for the study of the decays $D^+ \rightarrow \pi^-\pi^+\pi^+$, $D^+ \rightarrow K^-\pi^+\pi^+$, $D^0 \rightarrow K_S^0\pi^+\pi^-$ and $D^0 \rightarrow K_S^0 K^+K^-$. These parametrizations, where two-body hadronic final state interactions are taken into account in terms of unitary S - and P -wave $\pi\pi$, πK and $K\bar{K}$ form factors, are derived from different phenomenologically successful works (except for the $D^0 \rightarrow K_S^0 K^+K^-$ not tested yet) based on quasi two-body factorization approach.

5.2 Outlook

5.2.1 Some other studies on hadronic D decays

Within the factorization approximation and taking into account final state interaction, two-body hadronic D^0 , D^+ and D_s^+ decays have been studied in Ref. [48]. A reasonable agreement with the data, which show some large flavor SU(3) symmetry violation, is obtained. CP violating asymmetries are also discussed.

The authors of Ref. [49] explore consequences of constraints from CPT symmetry on three-body D decays. They simulate the $D^\pm \rightarrow \pi^\mp K^+ K^-$ decays and discuss correlations with measured $D^\pm \rightarrow \pi^\mp \pi^+ \pi^-$.

The impact of New Dynamics is studied in Ref. [50].

The authors of Ref. [51] perform an analysis of pure annihilation type diagrams for two-body $D \rightarrow PP(V)$ decays based on the k_T factorization. Their results agree with the existing experimental data for most channels.

5.2.2 Concluding summary

There is and there will be an impressive amount of high-quality hadronic-multibody decay data of D^0 , D^+ , D_s^+ . Improved models are and will be needed for extracting accurate informations from these data. In this brief review I described some available potentialities for constraining amplitude analyses in some of these charm decays. I have listed different effective hadronic formalism approaches. Detailed outcomes of a model, with combination of basic elements of weak interaction together with final state constraints in the framework of a Chiral unitary approach in coupled channel, have been reported.

I have described the diagrammatic-approach framework. It consists of a model-independent analysis, based on flavor SU(3) symmetry topological amplitudes, allowing to understand the relative importance of different underlying decay mechanisms. In this framework a global analysis of two-body D decays into a vector meson and a pseudoscalar meson has been presented.

A quasi-two-body QCD factorization model for D decays into three mesons and its recent application to $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ was shown to describe successfully the data. It suggests to go beyond

the superposition of Breit-Wigner amplitudes in the analysis of three-body decays with pions and kaons in the final state. It is advocated to use a phenomenological quasi-two-body factorization approach in which two-body hadronic final state interactions are fully taken into account in terms of unitary S - and P -wave $\pi\pi$, πK and $K\bar{K}$ form factors.

It could be interesting to see if the above phenomenological quasi-two-body factorization approach could be used in amplitude analysis of four-body hadronic D decays such as $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ recently analyzed by the CLEO Collaboration [52], $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ [53] and $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ presently studied by the BES III Collaboration [8].

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